Technical Note: High-Precision Spectral Analysis of the Local Edge-Flip Hamiltonian at $k = 16$

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Abstract: We extend the spectral analysis of 2+1D quantum gravity to level $k = 16$ ($N = 969$) using a local **Edge-Flip Hamiltonian**. We demonstrate that while this discretization leads to a more rapid gap collapse ($\gamma \approx 1.44$) than the global vertex model ($\gamma \approx 0.90$), it preserves the topological vacuum manifold of the torus (34 zero-modes). We conclude that while the existence of the gap is robust, the specific scaling exponent is discretization-dependent on coarse lattices, requiring further triangulation refinement to identify a universal continuum value.

I. Model Reconciliation and Universality

Note on Exponent Inconsistency: The discrepancy between the parent paper's exponent ($\gamma \approx 0.90$) and the current note ($\gamma \approx 1.44$) arises from the "stiffness" of the constraint discretization. The global vertex model enforces flatness across the entire manifold in a single "block" move, potentially overestimating the mass scale. Conversely, the Edge-Flip model implements a **locally resolved** discretization; by relaxing the constraints edge-by-edge, it allows for lower-energy excitations and a more rapid collapse of the gap as $\Lambda \to 0$. Importantly, this implies that neither exponent can yet claim to be the universal physical value. A true determination of γ requires refining both models toward the $N_{\text{tri}} \to \infty$ limit to observe if the scaling behaviors converge.

II. Consistency and Validation

The $k = 16$ results satisfy three critical internal validation criteria:

- **Topological Manifold:** Exactly **34 zero-modes** $(2 \times (k + 1))$, confirming the parity-doubled torus vacuum.

- **Geometric Symmetry:** A 3-fold degenerate excited triplet ($E_{34..36}$), reflecting the S_3 permutation symmetry of the theta-graph edges.
- **Numerical Floor:** Ground state at $\approx 10^{-15}$ ensures the operator is exactly positive semi-definite.

Table 1: Consistent Edge-Flip Scaling Data

Level k	Basis Dimension N	Zero-Modes	Numerical Gap Δ_{num}
3	20	8	0.252313
4	35	10	0.165412
6	84	14	0.086431
12	455	26	0.024561
16	**969**	**34**	**0.010145**

III. Statistical Regression

An Ordinary Least Squares (OLS) fit on the log-transformed data from Table 1 yields a numerical scaling exponent of $\beta = 1.88 \pm 0.06$ ($R^2 = 0.999$). Applying the physical mapping $\Delta_{\text{phys}} = \sqrt{\Lambda} \cdot \Delta_{\text{num}}$ where $\Lambda \propto k^{-2}$:

$$\Delta_{\text{phys}} \propto \Lambda^{(1+1.88)/2} = \Lambda^{1.44 \pm 0.03}$$

IV. Code and Data Availability

Repository: Codeberg / hamil-3-qg-pos-cosmo

- calc-for-k-16.py: Edge-flip matrix construction script.
- k-16-cal-results.txt: Console log for $N = 969$ basis generation.
- calc-eig-k-16.py: Eigenvalue post-processing and gap extraction.
- eig-k-16.txt: First 40 eigenvalues.